Progression Towards a Written Method for Multiplication

In developing a written method for multiplication, it is important that children understand the concept of multiplication, in that it is:

- repeated addition

They should also be familiar with the fact that it can be represented as an array

They also need to understand and work with certain principles, i.e. that it is:

- the inverse of division
- commutative i.e. 5 \times 3 is the same as 3 \times 5
- associative i.e. 2 \times 3 \times 5 is the same as 2 \times (3 \times 5)

YR

| Early Learning Goal: |
| Children solve problems, including doubling. |

Children are encouraged to develop a mental picture of the number system in their heads to use for calculation. They should experience practical calculation opportunities using a wide variety of equipment, including small world play, role play, counters, cubes etc.

Children may also investigate putting items into resources such as egg boxes, ice cube trays and baking tins which are arrays.

They may develop ways of recording calculations using pictures, etc.

A child's jotting showing the fingers on each hand as a double.  
A child's jotting showing double three as three cookies on each plate.
In year one, children will continue to solve multiplication problems using practical equipment and jottings. They may use the equipment to make groups of objects. Children should see everyday versions of arrays, e.g. egg boxes, baking trays, ice cube trays, wrapping paper etc and use this in their learning, answering questions such as ‘How many eggs would we need to fill the egg box? How do you know?’

Children should understand and be able to calculate multiplication as repeated addition, supported by the use of practical apparatus such as counters or cubes. e.g.

5 x 3 can be shown as five groups of three with counters, either grouped in a random pattern, as below:

or in a more ordered pattern, with the groups of three indicated by the border outline:
Children should then develop this knowledge to show how multiplication calculations can be represented by an array, (this knowledge will support with the development of the grid method in the future). Again, children should be encouraged to use practical apparatus and jottings to support their understanding, e.g.

5 x 3* can be represented as an array in two forms (as it has commutativity):

5 + 5 + 5 = 15

3 + 3 + 3 + 3 + 3 = 15

*For mathematical accuracy 5 x 3 is represented by the second example above, rather than the first as it is five, three times. However, because we use terms such as 'groups of' or 'lots of', children are more familiar with the initial notation. Once children understand the commutative order of multiplication the order is irrelevant).
End of Year Objective:
Write and calculate mathematical statements for multiplication using the multiplication tables that they know, including for two-digit numbers times one-digit numbers,

*Although the objective suggests that children should be using formal written methods, the National Curriculum document states “The programmes of study for mathematics are set out year-by-year for key stages 1 and 2. Schools are, however, only required to teach the relevant programme of study by the end of the key stage. Within each key stage, schools therefore have the flexibility to introduce content earlier or later than set out in the programme of study.” p4

It is more beneficial for children’s understanding to go through the expanded methods of calculation as steps of development towards a formal written method.

Initially, children will continue to use arrays where appropriate linked to the multiplication tables that they know (2, 3, 4, 5, 8 and 10), e.g.

3 \times 8

They may show this using practical equipment:

```
  x x x x x x x x
  x x x x x x x x
  x x x x x x x x
```

or by jottings using squared paper:

```
x x x x x x x x
x x x x x x x x
x x x x x x x x
```
As they progress to multiplying a two-digit number by a single digit number, children should use their knowledge of partitioning two digit numbers into tens and units/ones to help them. For example, when calculating 14 x 6, children should set out the array, then partition the array so that one array has ten columns and the other four.

Partitioning in this way, allows children to identify that the first array shows 10 x 6 and the second array shows 4 x 6. These can then be added to calculate the answer:

\[
(6 \times 10) + (6 \times 4)
\]

\[
= 60 + 24
\]

\[
= 84
\]

This method is the precursor step to the grid method. Using a two-digit by single digit array, they can partition as above, identifying the number of rows and the number of columns each side of the partition line.
By placing a box around the array, as in the example below, and by removing the array, the grid method can be seen.

\[
\begin{array}{c|c|c}
6 & 60 & 24 \\
\end{array}
\]

It is really important that children are confident with representing multiplication statements as arrays and understand the rows and columns structure before they develop the written method of recording.

From this, children can use the grid method to calculate two-digit by one-digit multiplication calculations, initially with two digit numbers less than 20. Children should be encouraged to set out their addition in a column at the side to ensure the place value is maintained. When children are working with numbers where they can confidently and correctly calculate the addition mentally, they may do so.

\[
13 \times 8
\]

\[
\begin{array}{c|c|c|c}
\times & 10 & 3 & 80 \\
8 & 80 & 24 & + 24 \\
\end{array}
\]
When children are ready, they can then progress to using this method with other two-digit numbers.

\[
\begin{array}{cccc}
\times & 30 & 7 \\
6 & 180 & 42 \\
\end{array}
\]

\[+ \quad 180 \quad 42 \]
\[
\frac{222}{222}
\]

Children should also be using this method to solve problems and multiply numbers in the context of money or measures.

Y4

End of Year Objective:
Multiply two-digit and three-digit numbers by a one-digit number using formal written layout.

Children will move to Y4 using whichever method they were using as they transitioned from Y3. They will further develop their knowledge of the grid method to multiply any two-digit by any single-digit number, e.g.

\[
\begin{array}{cccc}
\times & 70 & 9 \\
8 & 560 & 72 \\
\end{array}
\]

\[+ \quad 560 \quad 72 \]
\[
\frac{632}{632}
\]

To support the grid method, children should develop their understanding of place value and facts that are linked to their knowledge of tables. For example, in the calculation above, children should use their knowledge that \(7 \times 8 = 56\) to know that \(70 \times 8 = 560\).

By the end of the year, they will extend their use of the grid method to be able to multiply three-digit numbers by a single digit number, e.g.
When children are working with numbers where they can confidently and correctly calculate the addition (or parts of the addition) mentally, they may do so.

Children should also be using this method to solve problems and multiply numbers in the context of money or measures.
Children should continue to use the grid method and extend it to multiplying numbers with up to four digits by a single digit number, e.g.

\[
\begin{array}{c|cccc}
  \times & 4 & 000 & 300 & 40 & 6 \\
  8 & 32000 & 2400 & 320 & 48 \\
\end{array}
\]

\[
\begin{array}{c}
  32000 \\
  + \quad 2400 \\
  + \quad 320 \\
  + \quad 48 \\
\end{array}
\]

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and numbers with up to four digits by a two-digit number, e.g.

\[
\begin{array}{c|ccc}
  \times & 2000 & 600 & 90 & 3 \\
  20 & 40000 & 12000 & 1800 & 60 \\
  4 & 8000 & 2400 & 360 & 12 \\
\end{array}
\]

\[
\begin{array}{c}
  40000 \\
  + \quad 8000 \\
  + \quad 12000 \\
  + \quad 2400 \\
  + \quad 1800 \\
  + \quad 360 \\
  + \quad 60 \\
  + \quad 12 \\
\end{array}
\]

\[64632\]

When children are working with numbers where they can confidently and correctly calculate the addition (or parts of the addition) mentally, they may do so.

Children should also be using this method to solve problems and multiply numbers in the context of money or measures.
Once the grid method is fully embedded children to progress to formal written methods in the form of short and long multiplication.

**Short multiplication**

\[
\begin{array}{c}
45 \times 8 \\
\times 8 \\
\hline
360
\end{array}
\]

\[
\begin{array}{c}
345 \times 3 \\
\times 3 \\
\hline
1035
\end{array}
\]

**Long multiplication**

\[
\begin{array}{c}
24 \times 16 \\
\times 16 \\
\hline
384\quad 240 \\
\hline
3840
\end{array}
\]

\[
\begin{array}{c}
344 \times 23 \\
\times 23 \\
\hline
688\quad 2400 \\
\hline
8020
\end{array}
\]

End of Year Objective:

Multiply multi-digit numbers up to 4 digits by a two-digit whole number using the formal written method of long multiplication.
By the end of Y6, children should develop the method to be able to multiply decimal numbers with up to two decimal places, e.g.

\[
\begin{array}{c|cc}
& 4 & 0.9 \\
\hline
3 & 12 & 2.7 \\
\end{array}
\quad 12
\]
\[
\begin{array}{c|cc}
& 4 & 0.02 \\
\hline
3 & 12 & 2.7 \\
\end{array}
\quad + 0.06
\]
\[
\begin{array}{c|cc}
& 4 & 0.06 \\
\hline
3 & 12 & 2.7 \\
\end{array}
\quad + 2.7
\]
\[
\begin{array}{c|cc}
& 4 & 0.06 \\
\hline
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